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Thermoelectric-generator with linear phenomenological heat-transfer law

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Abstract

The performance of multi-element thermoelectric-generators, assuming heat-transfer irreversibilities which obey the linear phenomenological heat-transfer law $Q \propto (\Delta T^{-1})$, is studied in this paper by combining finite-time thermodynamics with non-equilibrium thermodynamics. The performance characteristics of the output power, efficiency and working electrical-current are described by numerical examples.

Keywords: Linear phenomenological heat-transfer law; Thermoelectrical generators; Performance analysis

1. Introduction

Several authors have applied non-equilibrium thermodynamics to the performance analyses of thermoelectric-generators [1–4]. They neglected the heat-resistance loss between the thermoelectric equipment and external heat-reservoirs, so there were limitations to the conclusions. In recent years, with the development of finite-time thermodynamics [5–13], some authors have applied it to the analysis of the thermoelectric-generator, and obtained some new findings [14–24]. The authors of [14–23] ana-

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lyzed the effects of the finite-rate heat transfer between the thermoelectric device and its external heat-reservoirs on the performance of the single-element thermoelectric-generator. In practice, a thermoelectric-generator is composed of many fundamental thermoelectric elements. It is a multi-element device. Chen et al. [24] investigated the characteristics of a multi-element thermoelectric-generator with the irreversibility of finite-rate heat transfer, Joulean heat generated inside the thermoelectric device, and heat leaks through the thermoelectric couple. In the analyses of [14–24], the heat transfer between the thermoelectric generator and the external heat-reservoirs was assumed to obey the Newtonian (linear) heat-transfer law $Q \propto (\Delta T)$.

Much work has shown that the heat-transfer law between the cycle and its external reservoirs affects the performances of conventional thermodynamic cycles strongly [25–32]. The purpose of this paper is to explore the performance of the thermoelectric generator with multi-element thermoelectric equipment by assuming that the heat transfer between the thermoelectric-generator and the external heat reservoirs obeys another linear law, i.e., the linear phenomenological law used in the irreversible thermodynamics $Q \propto (\Delta T^{-1})$. The output power versus working electrical-current curves and the efficiency versus working electrical current are given by numerical examples.

2. Theoretical analysis

Real thermoelectric-generators are composed of many thermoelectric elements, as shown in Fig. 1. Each element is composed of P-type and N-type semiconductor legs. Each thermoelectric element is assumed to be insulated, both electrically and thermally, from its surroundings, except at the junction–reservoir contacts. The internal irreversibility is caused by Joulean electrical-resistive loss and heat-conduction loss through the semiconductors between the hot and cold junctions. The Joulean loss generates internal heat I^2R, where R is the total internal electrical resistance of the semiconductor couple and I is the electrical current generated

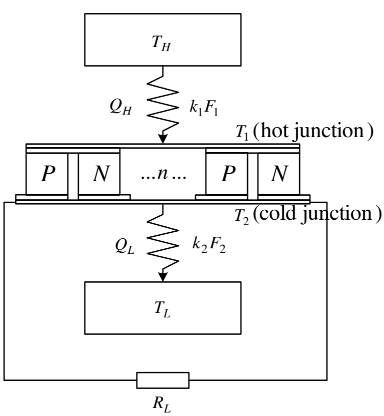

Fig. 1. Thermoelectric-generator.

by the couple. The conduction heat-loss is $K(T_1 - T_2)$, where K is the thermal conductance of the semiconductor couple, T_1 is the hot junction temperature, and T_2 is the cold junction temperature. The external irreversibility is caused by finite-rate heat-transfers. For a thermoelectric generator composed of n thermoelectric generating elements, the rate (Q_H) of heat transfer from the heat source T_H to the hot junction at temperature T_1, and the rate (Q_L) of heat transfer from the cold junction at temperature T_2 to the heat sink T_L are, respectively:

$$Q_H = n[\alpha I T_1 + K(T_1 - T_2) - 0.5I^2R], \tag{1}$$

$$Q_L = n[\alpha I T_2 + K(T_1 - T_2) + 0.5I^2R], \tag{2}$$

where $\alpha = \alpha_P - \alpha_N$, α_P, α_N are the Seebeck coefficients of the P- and N-type semiconductor legs for each thermoelectric element and n is the number of thermoelectric elements in the generator.

Assuming that the heat flux Q between the thermoelectric generator and external heat-reservoirs obeys the linear phenomenological heat transfer law $Q \propto (\Delta T^{-1})$ [25–32], the heat flux at the hot and cold junctions are, respectively:

$$Q_H = k_1F_1(1/T_1 - 1/T_H), \tag{3}$$

$$Q_L = k_2F_2(1/T_L - 1/T_2), \tag{4}$$

where k_1F_1 and k_2F_2 are the heat conductances (product of heat-transfer coefficient and heat-transfer surface area) of the heat exchangers between the hot and cold junctions of the thermoelectric generator and their respective reservoirs.

Combining Eqs. (1) and (3) yields

$$T_2 = (i+1)T_1 - \frac{k_1F_1}{nT_1T_HK}(T_H - T_1) - \frac{i^2}{2Z}, \tag{5}$$

where $i = \alpha I/K$ is the dimensionless electric current, and $Z = \alpha^2/(KR)$ is the figure-of-merit of the semiconductor element.

Combining Eqs. (2) and (4) yields

$$n(i-1)T_LKT_2^2 + \left(nT_1T_LK + \frac{i^2}{2Z}nT_LK - K_2F_2\right)T_2 + k_2F_2T_L = 0. \tag{6}$$

Substituting Eq. (5) into Eq. (6) yields

$$\begin{aligned} &n(i-1)T_LK\left[(i+1)T_1 - \frac{k_1F_1}{nT_1T_HK}(T_H - T_1) - \frac{i^2}{2Z}\right]^2 \\ &\quad + \left(nT_1T_LK + \frac{i^2}{2Z}nT_LK - K_2F_2\right)\left[(i+1)T_1 - \frac{k_1F_1}{nT_1T_HK}(T_H - T_1) - \frac{i^2}{2Z}\right] \\ &\quad + k_2F_2T_L = 0. \end{aligned} \tag{7}$$

The total power output (P) and the efficiency (η) of the multi-element generator are as follows:

$$P = Q_H - Q_L = n[\alpha I(T_1 - T_2) - I^2 R], \tag{8}$$

$$\eta = 1 - Q_L/Q_H. \tag{9}$$

3. Numerical examples and discussion

The performance of the thermoelectric-generator is analyzed by numerical calculations. In these, $T_H = 600$ K, $T_L = 300$ K, $\alpha = 2.3 \times 10^{-4}$ V/K, $R = 1.4 \times 10^{-3}$ Ω, $K = 1.5 \times 10^{-2}$ W/(Km), $k_1F_1 = 30{,}000$ W/K, and $k_2F_2 = 28{,}000$ W/K are set. For the fixed working electric-current, one can solve T_1 from Eq. (7). Substituting T_1 into Eq. (7) yields T_2. Substituting T_1 and T_2 into Eqs. (3), (4), (8) and (9) yields the heat fluxes at the hot and cold junctions, the output power and the thermal efficiency.

Figs. 2 and 3 show the characteristics of the output power versus working electric-current and the thermal efficiency versus working electric-current with different numbers of elements, respectively. It can be seen that for a fixed heat-conductance (k_1F_1 and k_2F_2) and a fixed number (n) of thermoelectric elements, there exists one optimal working electric-current $I_{P_{\max}}$ corresponding to the maximum power-output $P_{\max}$, and another optimal working electric-current $I_{\eta_{\max}}$ corresponding to the maximum efficiency $\eta_{\max}$. Both $I_{P_{\max}}$ and $I_{\eta_{\max}}$ decrease with the increase of number of thermoelectric elements. The efficiency decreases with an increase of number of thermoelectric elements. However, there exists a power extremal with respect to the number (n) of thermoelectric elements: there exists an optimal n corresponding to the maximum power-output. Therefore, there exists a double maximum power-output with the optimal working electrical current and the optimal number of thermoelectric elements. In the conventional analysis, without considering the heat-transfer effect,

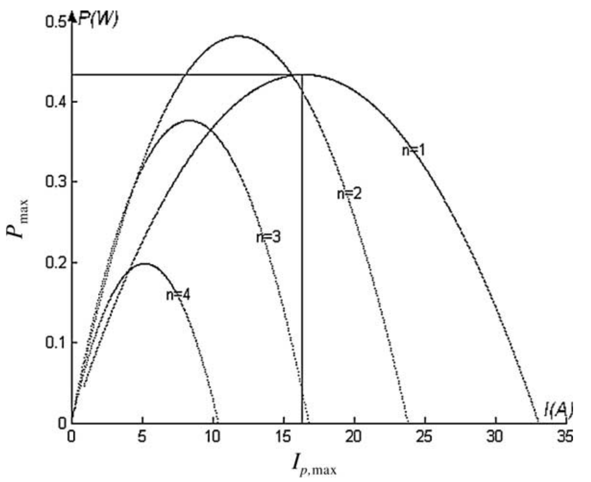

Fig. 2. Power versus electrical current and number of elements.

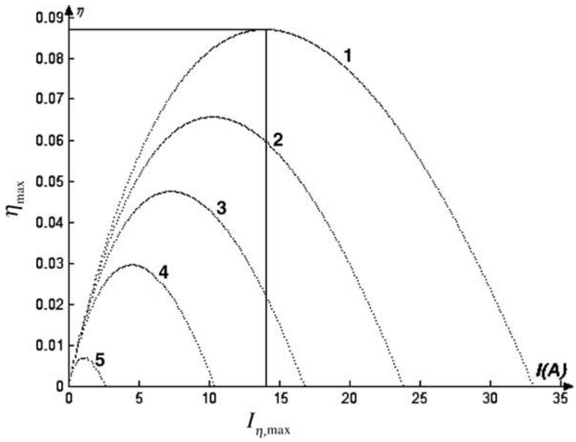

Fig. 3. Efficiency versus electrical current and number of elements.

the power output is a monotonic (linear) increasing function of n, and the efficiency is independent of n. Figs. 2 and 3 show that the heat-transfer irreversibility does affect the power output and the efficiency of the thermoelectric-generator. This effect must be considered in the performance analysis.

The power versus the thermal efficiency of the thermoelectric-generator is shown in Fig. 4. It can be seen that the P–η characteristic curve is loop shaped, and there is almost no difference between the maximum power point and the maximum thermal-efficiency point. Therefore, there exist two important points for the output power and the thermal efficiency of the thermoelectric-generator, i.e., there is an optimal working electric-current $I_{P_{\max}}$ with the maximum power output $P_{\max}$ and the corre-

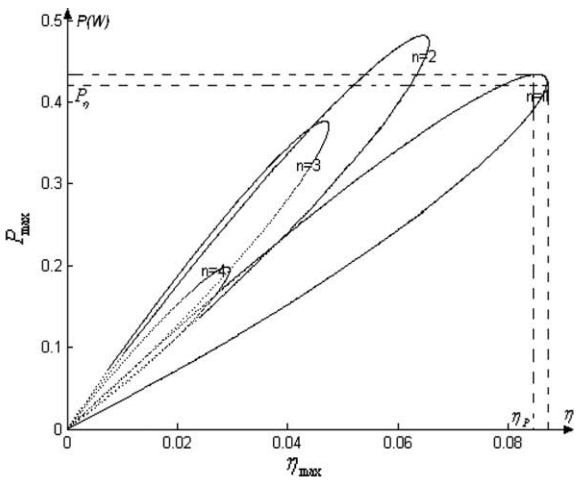

Fig. 4. Power versus efficiency with different number of elements.

sponding efficiency η_P: there is an optimal working electric-current $I_{\eta_{\max}}$ corresponding to the maximum thermal efficiency $\eta_{\max}$ and the corresponding output power P_η. This is consistent with those of a generalized irreversible Carnot engine theoretical model established by Chen et al. [31].

From the point of view of finite-time thermodynamic optimization (the compromise optimization between power output and the efficiency), the design parameter optimal region of the thermoelectric-generator should be:

$$P_\eta \leqslant P \leqslant P_{\max}, \tag{10}$$

$$\eta_P \leqslant \eta \leqslant \eta_{\max}. \tag{11}$$

These provide guidance for the selection of the parameters of the thermoelectric-generator.

4. Conclusion

The performance of the thermoelectric-generator assuming the linear phenomenological heat-transfer law has been studied in this paper. The result indicates that the heat-transfer law does affect the performance of the thermoelectric-generator, and affects the selection of the number of the thermoelectric elements directly. So the optimal electrical current and the optimal number of thermoelectric elements must be chosen as a result of a comprise optimization between the power output and the efficiency in order to obtain the highest power output and thermal efficiency.

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